Mathematics and Problem Solving

Lecture 8.1

Intro to Proof

Terminology

- **Proof** - A mathematical argument that something is true

- **Claim:** If m is an even number, m^2 is an even number

- **Conjecture:** If m is an even number, m^2 is an even number
- **Conjecture** – A mathematical claim that is intesting enough to try and prove.

- **Conjecture:** If m is an even number, m^2 is an even number
  - $m = 2k$
  - $m^2 = (2k)(2k)$
  - $m^2 = 4k^2$
  - $m^2 = 2(2k^2)$

- **Conjecture:** If m is an even number, m^2 is an even number
- **Proof:** Let k be a number such that m = 2k. Thus, $m^2 = 4k^2$. Which we can factorise as $m^2 = 2(2k^2)$. Therefore, m^2 is an even number. ∎

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- **Lemma** – a proposition proved as part of proving a theorem
- **Corrolary** – a theorem that follows directly from proving another theorem

- **Informal proof** – most proofs, rely on a mathematician's common sense
- **Formal proof** – expressed in a formal language
  - Derive new true statements from known true statements

Modus Ponens

- Law of reasoning
  - If $P \Rightarrow Q$ and P, then Q
- Familiar from propositional logic

- **Premise** 1: *If I like chocolate, I will eat the cake*
- Premise 2: *I like chocolate*
- **Conclusion**: *I will eat the cake*

- <string1>
- <string1> → <string2>
- <string2>

Direct Proof

- If we want to prove $P \Rightarrow Q$
  1) We assume P
  2) Show that Q logically follows from it

Example

- **Theorem:** If n is odd, then n^2 is odd
- **Proof:** Assume n is odd
  - $n = 2k+1$
  - $n^2 = (2k+1)^2$
  - $n^2 = 4k^2 + 4k + 1$
  - $n^2 = 2(2k^2 + 2k) + 1$
  - Therefore n^2 is odd ∎

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  - Therefore n^2 is odd ∎ *(Q)*

Summary

- **Conjectures** are claims we want to prove
- **Theorems** are claims we have proved
- **Direct Proof** is a way of proving $P \rightarrow Q$ by assuming P and showing that Q follows
- If m is even, $m = 2k$
- If m is odd, $m = 2k + 1$