Implication and Equivalence

Implication ($\Rightarrow$)

- Implication is a conditional
  - e.g. “if I am drinking tea then I am happy”
- **However**, when the antecedent is false, the statement becomes **vacuously true**

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Vacuous Truth

- Vacuous truth is a weird idea
  - The statement “all dogs on the moon are blue” is (vacuously) true
- In an implication, there is an antecedent and a consequent
  - If **antecedent**, then **consequent**
- If the antecedent is false, the statement is always (vacuously) true

Equivalence ($\iff$)

- An equivalence is true when **both** truth values are **the same**
  - e.g.this is true:
    - “I am wearing socks” $\iff$ “The sun rises in the east”
  - This is false:
    - “I am wearing socks” $\iff$ “The UK is in the southern hemisphere”
- Often read “if and only if”

p	q	$p \iff q$
T	T	T
T	F	F
F	T	T
F	F	T

Tautology

p	$p \iff p$
T	T
F	T

Contradiction

p	$p \iff \neg p$
T	F
F	F

Contingiency

p	$\neg p$
T	F
F	T

Precidence

- Precidence in Proposional Logic
  1. Negation $\neg$
  2. Conjunction $\wedge$
  3. Disjunction $\vee$
  4. Implication $\Rightarrow$
  5. Equivalence $\Longleftrightarrow$
- For example $\neg p \wedge q$ means $(\neg p) \wedge q$ and not $\neg(p \wedge q)$.

Summary

- Logical Operators
  - Implication $\Rightarrow$
    - Vacuous Truth
  - Equivalence $\Longleftrightarrow$
- Tautology
- Contradiction
- Contingiency