

Propositional Logic

If statements

- We often use if statements in programming

if a == b:

 #do something

- Sometimes we need something more complex

if a > 0 **and** a < 10:

 #do something

if b < 0 **or** b > 10:

 #do something

Simplifying

- Sometimes there is a simpler form that is **logically equivalent**, for example
 - if $\neg(\neg(a \text{ or } c) \text{ and } (a \text{ or } c))$ and $(a == c)$:
 - if $(a == c)$:
- How do we know when we can do this?
- How can we prove it is valid?
 - Propositional Logic

Logical Operators

- The rules that Python and other programming languages use for **logical operators** come from a branch of maths called **propositional logic**.

and = \wedge

or = \vee

not = \neg

- Understanding propositional logic will help you use them well

What is Logic?

- In this context, logic refers to a specialised branch of mathematics
 - Used to study arguments
 - Used to understand reasoning
 - Used to look at premises and conclusions
 - And whether conclusions can be truthfully drawn from premises

Propositional Logic

- The kind of logic we are going to study here is called **propositional logic**
 - Or propositional calculus
- It deals with stringing together expressions called **propositions**
 - Which can be either true or false
- And determining whether combinations of those propositions are true or false

Propositions

- Propositions are statements that are either true or false
 - I am wearing socks
 - $12 + 1 = 13$
 - The sun rises in the east
- These are also called **atomic propositions** because their truth or falsity is not dependant on any other proposition.

Truth Values

- A proposition p must be **either** true or false
 - Often written T and F

p
T
F

Negation

- $\neg p$ is the **negation** of the proposition p
- Let p be the proposition 'It is raining outside'
 - $\neg p$ = It is not raining outside

p	$\neg p$
T	F
F	T

Compound Propositions

- $\neg p$ is a **compound proposition** because it combines:
 - other propositions (p); and
 - logical operators (\neg)
- We can also use letters to stand for compound propositions, e.g. $q = \neg p$

Summary

- Propositional logic works with propositions
 - Atomic (e.g. $p = \text{“It is raining outside”}$)
 - Compound (e.g. $q = \neg p$)
- Each proposition is either **true** or **false**
- We can determine whether a statement is true **relative to the atomic propositions it contains**