

Introducing Sets

Films

- Lets say we want to decide what film to watch.
 - We have a bunch of different people. Each person has films they like and films they have seen before.
- What film do we watch?
 - It's got to be a film we own.
 - Each person only wants to re-watch a film if they like it.
 - But they're happy to watch any film for the first time

Films

- Problem:
 - It's getting kind of complicated to describe.
- Bigger Problem:
 - Imagine we were writing the specification for a Film recommendation system.
 - Or responsible for coding one from such a specification
- We want to be able to describe it as precisely as possible.
- Formal notation using set theory will help here!

Introducing Sets

- Sets are collections of things
 - Elements
 - Other Sets
- Unordered
- Unique
- E.g. Numbers, Countries, Films

Set Theory

- There are various different branches of maths
 - Usually, each of these branches deals with some special problem
 - Or some special kind of data
 - Eg. numbers (arithmetic)
 - Or truth values (logic)
- Set theory deals with operations on sets. Rules and notation for manipulating them

Definition by Extension

- There are two ways to define a set.
- The first is simply to list the elements.
 - $A \triangleq \{1,2,3,4\}$
 - $B \triangleq \{\text{Up, Down, Left, Right}\}$
 - $C \triangleq \{\text{banana, custard, yoghurt}\}$
- You will see both $=$ and \triangleq used

Intensional Definition

- The other way is to give the conditions that describe the elements of the set without ambiguity
 - Films $\triangleq \{ f \mid f \text{ is a film that we own} \}$
 - in maths, \mid often means ‘given that’
- Two ways to define a set with the same elements:
 - $A \triangleq \{1,2,3,4\}$
 - or $A \triangleq \{y \mid y > 0 \text{ and } y < 5\}$

The Empty Set

- An important set to know is the set with nothing in it. It's called the empty set.
 - Uses the symbol \emptyset
 - $\emptyset \triangleq \{ \}$
- Note that $\emptyset \neq \{\emptyset\}$
 - $\{\emptyset\}$ is a set containing the empty set, not the empty set itself!

Singletons

- A singleton is a set containing only one member
 - $\{1\}$
 - The set of prime numbers between 6 and 10

Familiar Sets

- $\mathbb{N} \triangleq \{0, 1, 2, 3, \dots\}$ (the 'natural numbers')
- $\mathbb{Z} \triangleq \{\dots -3, -2, -1, 0, 1, 2, 3 \dots\}$ (the 'integers')
- \mathbb{R} (the 'real numbers')

Summary

- A set is a collection of elements
- The elements of a set have no order
 $\{a, b, c\} = \{b, c, a\}$
- Elements must be unique
 $\{a, a, b\}$ is not a valid set, where $\{a, b\}$ is